STEP Tips

A.T.

March 2020

1 Exam Technique

- Don't be afraid of spending 5-10 minutes at the start of the paper **selecting questions**. Recent papers have longer questions and it's far better to do several almost-complete attempts than to try the first parts of several questions and leave it there.
- In general *very little credit* will be given for only doing the very first part of a question, so unless you've already done 4-5 solid solutions don't just look through and try little bits of questions because you won't get many marks.
- Often, questions with more question parts (e.g. 4 parts) that *look long* can actually be easier. It's also a good idea to try questions that give you *lots of guidance* towards reaching the solution, so you spend less time trying to spot the trick. Shorter looking questions are often harder.
- When selecting questions at the beginning of a paper, try to identify the ones where it's more obvious how to proceed.
- 5-6 good but incomplete solutions can get you a 1, but it's important to ensure you do at least 2-3 complete solutions - don't worry too much about time and just try to keep working towards the solution.
- It's often safer to go for questions that explicitly state what you need to show ("show that..."), rather than ones that just describe what you have to do ("find an expression for...") since if you make a mistake in these then you won't know about it until you check.
- When selecting questions, look not only for ones that give lots of guidance, but also for questions with lots of *accessible marks*. For example, a question might have a part where you just have to differentiate an expression twice, which gives several marks that you can get without having to spot many tricks.
- *Never* rely on fragmentary attempts at questions to bring up your score. You need to identify at least 5 questions where you can get a good number of marks. Most of these should have almost all the marks accessible, and the worst of these should still have about 1/2 to 2/3 marks accessible. 4 good attempts plus two fragmentary attempts may not be enough. Remember the importance of question choice.
- Some questions may ask you to make a substitution where you have to choose suitable values. An example is "by using a substitution k/x for a suitable choice of k, evaluate I". Unless you can immediately spot what the value should be, try to de-prioritise these questions.
- If you've already spent 30-40 minutes on a question and don't think you'll be able to finish it (which may happen e.g. if the algebra gets overcomplicated) then state your intentions by writing down what you'd do next before moving on - you may just score some method marks.
- Question 1 is usually designed so almost all candidates will be able to score some marks, so it tends to be a perfect example of a good question to choose. Question 2 is usually also fairly accessible, but to a lesser extent than 1.

2 Using Earlier Parts to Help with Later Parts

These questions usually fall into a few categories.

- A substitution transforms an expression in a later part into an expression that's very similar to one in the previous part.
- You are given a method to use to answer the first part. The second part doesn't simplify down to something in the first part, but you have to apply the same method (perhaps with a small adaptation, e.g. a linear combination) to the second part.
- If you are asked to work out the first few values of a sequence, this may be useful later for spotting a pattern in the first few terms.
- If you have to prove an equation is true in the first part, and then the second part has a very similar looking equation to prove, then it's probably the original equation with a small change such as $c = a + b$.
- If the second part looks completely different to the first part, don't worry too much - it's probably just a straightforward substitution to turn the first part into a different form.

3 Approaches

- It is almost certain that earlier parts of a question should be used to complete later parts. If "hence or otherwise" is used, it's almost *always* easier to use "hence".
- If you are asked to "deduce" something, then usually not much more working is required, but it must still be a fully logical deduction.
- If you are given a substitution for the first part of a question, and then have to answer a second question part with no guidance given, then try a slight modification to the original substitution such as a linear combination.
- Try and factorise expressions if possible. Products are more useful than sums in determining the signs of unknowns. This can also be useful if the unknowns are primes or integers.
- Try not to overthink the stem or part (i) of a question - it's probably not really that difficult.
- Suppose you're doing a question and you can see exactly what you need to do, but it looks like the algebra will get extremely complicated. Always check to see if there's a faster way than what you had in mind. Sometimes getting stuck in a mess of algebra can leave you unable to finish the question.
- If you're given (or have proved) a result with a constraint such as $|x| < 1$ or $y > 0$, you need to check that this constraint is satisfied before applying the result to a new situation, and justify it in your solution to ensure you don't lose a mark for rigour.

4 Sequences, Series and Recurrences

- If asked to find a general formula for a sequence, try looking at the first few terms to guess a pattern and then prove it by induction.
- If you aren't given enough terms to be sure of the pattern (e.g. $2, -2, 4, ...$) then look at how the terms are generated to help you work out a general formula.
- Try to use a method-of-differences approach to simplify sequences. For example you can rewrite $\ln(\frac{k+1}{k})$ as $\ln(k+1) - \ln k$ and then most of the terms cancel.

- When simplifying an infinite series using method of differences, you must justify that as n goes to infinity, the nth term goes to zero, so cancelling is allowed.

- If you have to evaluate a series where some power such as 2^{-n} has been multiplied by, then it's usually just a particular choice of x that you have to determine.

5 Recurrence Integrals

Often you will be asked to find I_{n+1} in terms of I_n. There are two main possible approaches.

- **Approach 1.** Use integration by parts on either I_{n+1} or I_n. Identify a part of the integrand that you know how to integrate to choose as the v' term. Then you might end up with an expression that simplifies to what you want to show.

- **Approach 2.** Combine both I_n and I_{n+1} over the same denominator so you can add/subtract them to combine them into one integral, which may be easier to evaluate. Example:

  (Notice that in this example the last integral isn't easier to evaluate than the original I_n but it's just to illustrate the process.)

$$I_n + I_{n+1} = \int_0^1 \frac{1}{(1+x)^n}\,\mathrm{d}x + \int_0^1 \frac{1}{(1+x)^{n+1}}\,\mathrm{d}x = \int_0^1 \frac{1+x}{(1+x)^{n+1}}\,\mathrm{d}x + \int_0^1 \frac{1}{(1+x)^{n+1}}\,\mathrm{d}x = \int_0^1 \frac{2+x}{(1+x)^{n+1}}\,\mathrm{d}x$$

6 Graphs

- When sketching a graph:
  - First check for what values of x the function is defined.
  - Work out what happens as x goes to $+\infty$ and $-\infty$ or as x approaches any asymptotes.
  - Find where the graph intersects the coordinate axes.
  - Work out where any turning points or inflection points are. You may not *have* to use differentiation unless the information you already have is insufficient. Awkward differentiation should be a last resort.
  - Check the graph is vertically in a reasonable range by checking particular points.

- If you have to sketch several graphs in one question that have similar looking equations, it's possible that some of them are just transformations of others.

7 Algebra

- A polynomial in more than one variable (e.g. x and y) can often be factorised using a similar approach to factorising a single-variable polynomial.

  Example: $x^3 - 4x^2 - x + 4 = (x-1)(x-4)(x+1)$

  $x^3 - 4x^2y - xy^2 + 4y^3 = (x-y)(x-4y)(x+y)$

- If there are equations, graphs or integrals involving the modulus function, split it up/consider the different regions of the graph or the different ranges that x could fall into.

- When finding the maximum or minimum value of a function within a certain interval, remember to consider the values of the function at the endpoints of the interval as well as the stationary points in the interval.

- If an expression needs to be rearranged or substituted and the algebra seems extremely long-winded, there is usually an easier way.

  Example: $x^2 - 4x + \sqrt{2x^2 - 8x - 3} - 9 = 0$

  Substitute $y = \sqrt{2x^2 - 8x - 3}$

  You don't then have to solve the second equation to get x in terms of y and then sub it back into the first equation in place of each x. You can just notice that $x^2 - 4x$ from the first equation is the same as $\frac{1}{2}(y^2 + 3)$ and put that expression straight in to get an equation in just y.

- If you're working with an equation with very large numbers or coefficients in it, try writing large numbers as products of smaller numbers to make cancelling easier. Try to avoid having to do long multiplication.

- If you have an algebraic identity, you can equate coefficients to find the value of some quantity.

- Remember that $\sqrt{x^2} = |x|$, NOT just x.

- The roots of polynomials results can be applied in questions not specifically about algebra - particularly coordinate geometry. If you're asked to find an expression for $p + q$ or pq then it's likely that you can use these results somehow.

- If you end up with an expression that is undefined for the specific value given (such as a differential equation solution), then try to rewrite it in a form that is defined by getting rid of zero denominators or taking limits.

- Always try to simplify the algebra, especially in mechanics, so you're less likely to make mistakes.

- Look for symmetry in questions with a heavy algebra or polynomial related aspect. If an equation is symmetric in a, b and c and $a + b - c$ is a factor of the expression, then $b + c - a$ and $c + a - b$ must also be factors by symmetry.