Oxbridge Maths Preparation Guide

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How to use the resources

Hi. I'm Vishal, a first year maths PhD student at Oxford. I did my undergraduate and masters at Cambridge. Hopefully the resources in this folder are useful - they are what I used to prepare for my interview. A lot of the resources here were from a folder created by Izaac Mammadov. A lot of the resources in the STEP section were collected by the people who made the STEP megapack and there's accompanying threads on TSR about STEP, this one usually has a new version every year. The advice is based on my experience of having done MAT, STEP and the interview in 2019/2020, along with my experience in hosting workshops for STEP & the interview and having marked STEP in 2024.

Many Oxbridge colleges post videos online of interviews - they might be quite useful (Churchill has one). Oxford University has sessions run by Dr. James Munro, Admissions director at Oxford on youtube. He ran some livestreams for MAT too. Some useful links with more questions can be found on the websites of the universities for example at Oxford.

The quality of resources varies massively here. Also, you don't need to get through all of them - I didn't. There are some resources which I've found after my interview. I'll use M for Maths people and CS for Computer Science. I would rank the usefulness as follows:

- 1. Mr Bowler's Interview Problems. There's about 150 of these; they're challenging, fun, and cover pretty much all of the major topics which come up in interview like recurrence relations, graph sketching, and some other ideas on the side like the extremal principle (M, CS). They're in the Interview problems folder - called TBO problem-solving booklet (in interview questions).
- 2. Trinity College Cambridge Tests. Use them like mocks these will likely be as harder than a typical Oxbridge maths interview because you're not expected to complete all of them. The Downing test is also useful but it's quite short. The Trinity tests are 1 hour long, while the Downing test is 30 minutes long. There's also an Oxford interview one in the same folder.
- 3. **CSAT papers:** Even though it's specifically computer science, maths people can find it useful too as the problems here are of high quality and challenging. There are solutions available on the website and a 5th paper is available there too (beyond the four in the folder). If you find them useful, leave a testimonial as they're of great importance to Dr. Roman and his team who keep CSAT running.
- 4. STEP Foundation modules/STEP 1. Helps with STEP preparation and learn-

ing some problem-solving skills (M). I recommend doing all of the STEP 1 ones and Calculus/Graph sketching at least from STEP 2 before the interview. It's a good idea to get through these in the summer before year 13.

5. Olympiad problems (Maths, Physics, Informatics are useful) can be helpful. I know people who've been asked Olympiad problems in interviews. For maths, it's definitely worth being aware of the Pigeonhole principle, Modular arithmetic, Quadratic/Cubic residues, and double counting. Many handouts for these can be found online or in books like *The Art and Craft of Problem Solving* by Paul Zeitz or *Problem Solving Tactics* by A Di Pasquale and others. Brilliant is a brilliant place to find articles to learn about these things, as well as Art of Problem Solving (AOPS) for some learning and problems too. Here are some resources on the Pigeonhole principle and Double counting.

There are many Olympiad websites online with lots of good resources (like Evan Chen's) and the websites here are all quite good. Some of these topics like Pigeonhole are on Oxford's maths club. If you want to just go for some Olympiad problems, do BMO1 and BMO2 questions. (M,CS,P) Siklos book - Older STEP questions which have hints, full solution, and suggestion. As always try a problem for a while before checking any hints or solutions!

- 6. **Siklos book** Older STEP questions which have hints, full solution, and suggestions. As always, try a problem for a while before checking any hints or solutions!
- 7. MAT question 5s You'll likely do these papers if you're doing Maths or Computer Science anyway, but the older papers aren't very useful except for the question 5s; these are on random topics and can be like an interview problem. The papers are on the Oxford website and some of the older ones are here with solutions, a TSR thread. The old MAT papers are generally significantly easier than the current ones and questions 6/7 didn't exist. (Maths/CS)
- 8. TSR problems Some good, some bad. I've edited it to be more readable in the drive. I don't really recommend these unless you've already covered a lot and have nothing else to do. I don't think that will happen though when I prepared, I spent about two months preparing and didn't get through all the dedicated interview resources.

Sorry physics people, but outside of PAT papers, Physics Olympiad, and possibly STEP mechanics for some maths-style physics, I'm not too sure. I'll update this folder as I find anything for any subject.

For CS people, I'm not sure how much this helps with the interview, but it may be useful (or enjoyable) to do some competitive programming, solving (usually mathematical) programming problems with an emphasis on optimisation - it's both about solving the problem and making it run fast enough. Project Euler has many mathematical problems which need to be solved via programming, along with the British Informatics Olympiad.

Interview advice

This is mostly about Cambridge as that's where I studied, more will come about Oxford as I find out more - they are broadly similar except at Oxford they usually have 3 or

4 interviews spread across two colleges; the one you chose and one assigned to you. At Cambridge, some colleges have tests followed by an interview discussing the answers (e.g., Downing, Trinity, Kings), while others usually have two interviews, sometimes with prereading (e.g., Caius) or split into applied/pure (e.g., St John's), where applied can include topics like differential equations and vectors.

For both universities, interviews can be focused on one topic or a series of unrelated questions.

Some specific and often different to A level topics which can appear:

- 1. Quadratic residues are useful for questions like solving $x^2 + 4y^2 = 4643$ for positive integers x, y, knowing that a square modulo 4 is 0 or 1 simplifies the problem.
- 2. Another useful skill is graph sketching. They often ask about graphs because it showcases a wide range of skills and requires assimilating a lot of information. My director of studies at John's told me this is his favourite type of question to ask because of these reasons and that the analysis before drawing the graph is really the important part, over the actual sketch, especially in the context of online interviews. Some examples of graphs include $y = \cos(\cos(x))$ and $y = \tan(1/x)$. You can practice these with MAT papers and the STEP 2 module on curve sketching.

Strategies for graph sketching include thinking about specific values that are easy to evaluate, asymptotes, behaviour at 0, behaviour at infinity, using derivatives for stationary points, increasing/decreasing intervals, and points of inflection. You should also know about $y^2 = f(x)$ graphs.

It's always worth having an answer to 'Why do you like -subject-'? I got asked it in both my interviews.

Some advice I got from others, make sure to speak aloud about what you're thinking which allows them to assess you. Also, make sure you listen carefully to what the interviewers say. It's ok to be silent for a bit if you need to think and it's ok to get something wrong, the interviewers will help push you in the right direction to solve the problem. They want to see how you think so you won't be expected to get something wrong but instead have to stop and reason about the problems given.

At the end of it all, don't worry about the interview and instead think of it as an opportunity to do some interesting maths with people who want to encourage you to think hard on new problems. Good luck for your interviews and I hope you have fun!

MAT

This is relevant to the old style of MAT papers, 2023 and before. This section will be updated for the new papers once they've started to come out. I also have a youtube video on this.

For MAT, past papers are what I recommend. The ones from 2007 and onward reflect the current style better, but if you want more practice, you can use the old papers. However, they are generally easier. Starting with earlier papers is probably the best way to go, as the difficulty generally increases with a few spikes. You should leave a few years of papers

as mocks to emulate the real test. The questions follow more or less the same structure each time:

- Q2: Sequences/Algebra
- Q3: Graphs & Calculus
- Q4: Geometry
- **Q5:** More random but usually combinatorics/recurrence relations, or like Q7 with constructions based on given rules.

(I don't know much about Q6/7 except Q6 is generally logic, and Q7 is generally some sort of algorithmic process.)

Long questions

Identifying the topic like this means if you struggle on a certain question, you can go to the STEP database and search for harder questions on the same topic. If you can do STEP ones consistently, you likely can do MAT ones consistently too, as STEP is harder.

- Q2: Sequences. Try small values to spot any patterns or useful properties to help generalise. Induction is a good strategy for sequences, especially if there's some property you spot and want to prove in general. There's generally a lot of algebra in these questions too.
- Q3: A lot of the skills for graphs are helpful here. It's useful to be aware of things like even/odd functions and symmetry under reflection (2013 Q3). A useful thing to note is that the trapezium rule under/overestimates based on convexity/concavity, which is also useful for MCQs.

Overestimates and underestimates

In general, when the gradient of the graph is increasing over the given interval the area given by the Trapezoidal rule will be an overestimate of the actual area.



When the gradient of the graph is decreasing over the given interval the area given by the trapezium rule will be an <u>underestimate</u> of the actual area.



Q4: This is about geometry. It's good to be aware of all the circle theorems for this, and you'll be using a lot of areas of triangles and Pythagoras. It can sometimes be useful to draw lines on diagrams (including given ones), such as in 2013 Q4. There's also coordinate geometry, which works similarly but with things like perpendicular

lines having gradients that multiply to -1. Similar triangles also appear here, so it is worth being able to recognise them. When they exist, they can help simplify problems a lot (2016 Q4). Symmetry helps a lot in these problems too.

Q5: These questions are a bit more random. They're typically not on content that is on the A-level syllabus, so they feel like interview-style problems – they want to see how you think. Common elements are usually sequences, recurrence relations, and combinatorics. They sometimes give you rules to construct things (2014 Q5) or recursive functions (2019 Q5), which leads nicely into recurrence relations.

To form recurrence relations, think about the problem combinatorially (the usual technique you'll need in MAT is called committee selection) and break it down into cases. A good example is MAT 2014 Q5. This technique is very similar to double counting, where you count the number of things in two different ways to show that the two expressions are equal. There are good examples here with a part two.

For first order recurrence relations e.g. $x_n = 3x_{n-1} + 4$, solve by guessing $x_n = \lambda^n$ and adding constants where necessary or by adding 2 and making it geometric.

This website has a few good examples of problems where you need to break it down like MAT Q5s, such as the Tower of Hanoi and the Fibonacci rabbit problem. Another one is 'How many ways are there to fill a $2 \times n$ rectangle with 2×1 dominoes?'

Yet another good problem is **Putnam 2001 A2**: You have coins C_1, C_2, \ldots, C_n . For each k, C_k is biased so that, when tossed, it has probability $\frac{1}{2k+1}$ of landing heads. If the n coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of n. These questions might feel unfamiliar for a while, but with enough practice, they will become manageable.

Q6: I did only Q1-5 as I did MAT for Imperial maths but have had a look and thought about some Q6 and 7s - these are my thoughts:

In older papers, Q6 used to be logic puzzles, but in more recent papers, they've changed a fair amount. In logic puzzles, it's very much about going through all of the different cases and ruling out what's impossible to get to the conclusion (however improbable). In recent years, Q6 seems to often be about a procedure/algorithm. They introduce some concepts and want you to work with specific simple cases, and then it becomes more general as the question goes on. These smaller cases will help you get used to the more general process they want you to understand.

Q7: This is more consistent than Q6. They typically give you rules which define something, and you need to apply these rules, like axioms. A good example is 2013 Q7 where it wants to construct words based on some rules. In this way, there's overlap with other questions – Q5 and Q6. Q5 in particular, with things like the Martian Classical Period and 2018 Q5, is really similar. Like Q6, you generally work with a few simpler cases and then build up to more general/more complex cases. Just like those two questions, improving problem-solving skills is all you can really do. For those not doing Computer Science, doing Q6/7 might help with Q5 if you're struggling a lot on it.

For both Q6/7, the STEP foundation module warm-downs sometimes have logic-

style questions (especially like the old logic puzzle-style Q6), such as Assignment 12's warm down. You can see what the warm-down is in the description of the assignment – it's probably best to bother with only the ones which are relevant, but if you have time and find it fun, you can do all of them. The Oxford Computer Science department recommends this for Q6/Q7.

Regarding old MAT papers (1996–2006), there's not much point in doing them as they're generally too easy to be helpful – I remember worrying about not being able to do them as I discovered them a week before my exam and I felt like I was suddenly a lot less prepared! The most useful thing in these are the question 5s as there's not much like them in other things like STEP. MAT 1998, 2002, 2003, 2004, and 2006 Q5 seem to be the most relevant ones.

Multiple choice

This section now has a lot more importance with the new changes to the MAT. There are extra multiple choice questions on the website and the TMUA is entirely multiple choice so will be helpful in preparation, though it has some content and style differences.

1. You can avoid the working out a lot of the time by substituting certain values to eliminate certain cases (2019 Q1B), especially for long general calculations like integrals or expansions (2014 Q1G). Another good thing to do when direct values aren't easy is to consider extreme cases, such as x = 0 and when $x \to \pm \infty$ (2014 Q1D). This idea was also mentioned above in the part about graph sketching under interview advice.

This is really good for multiple-choice questions about graphs – usually all options can be eliminated via substitutions and some properties of the graph, like where it's positive/negative, etc. (2016 Q1E, 2015 Q1G, 2014 Q1B). Similar ideas work for comparing sizes of integrals, finding lower/upper bounds (2017 Q1J), a useful inequality is

$$\left| \int_{a}^{b} f(x) \, dx \right| \le (b-a) \max_{a \le x \le b} |f(x)|$$

Also for integrals, be aware of the reflection (or King's property): The idea behind this is very similar to the one in 2016 Q3 where the integral is the same because you're integrating over two intervals where the integral takes the same values but this holds generally.

$$\int_{a}^{b} f(x) dx = \int_{a}^{b} f(a+b-x) dx$$

and that if you reverse the limits on integrals, the sign changes

$$\int_{a}^{b} f(x) \, dx = -\int_{b}^{a} f(x) \, dx$$

2. Graphs are also a good way to investigate integrals/areas when it's hard to directly evaluate them (e.g., 2015 Q1F).

- 3. For Trigonometry, remembering identities (like $\sin(\pi/2 x) = \cos(x)$) and knowing graphs can help for finding numbers of solutions (e.g., 2015 Q1E). Specific values apply here too.
- 4. For floor function integrals, you can either substitute values or draw the graph and consider it as the sum of the areas of rectangles (e.g., 2013 Q1J).
- 5. For questions about comparing the values of expressions involving logarithms, it's much easier to deal with powers than logarithms, so convert them (e.g., 2015 Q1J). They generally ask which is the biggest, so compare them to the one you think is the largest.
- 6. For Sequence questions in general (not just in MCQs), if you're struggling, it's always a good idea to test a few small values to see if you can find a pattern or something relevant (e.g., 2017 Q1C, 2016 Q1G).
- 7. Sometimes questions ask about minimum values; these are often just quadratics, and completing the square or solving them can make the problem easier to deal with (e.g., 2014 Q1E, 2016 Q1D). Circles are also another good place to complete the square.
- 8. A particularly relevant graph sketching tip for MAT is that to determine the number of solutions of a cubic, it's sufficient to consider the signs of the turning points. If they're both the same sign, then there is only 1 root; if not, there are 3 roots. (You might want to draw graphs of the cases to see why this is true.)
- 9. A good resource if you run out of MAT material is DrFrost Maths, although I haven't personally used it, some people recommend it.

I once did a livestream on the STEP Discord server about MAT, here they are. The quality of the videos is probably quite poor though.

- Vishal Mat talk (1 out of 2).mp4
- Vishal Mat talk (2 out of 2).mp4

STEP

For STEP, much of the advice about past papers is similar to that for MAT. I have a youtube video on this. Don't worry about starting preparation late; most people start after they receive their offer in January. For interview preparation, I recommend tackling STEP 1 with resources such as the foundation modules available at Maths Foundation Modules. If you find the warm-ups too dry, you can skip straight to the questions. If you can solve them, move on; otherwise, work through the preparation materials. The warm-downs are generally just interesting problems to think about and can be useful for the interview. I recommend working on these over the holidays of Year 12.

Modules for STEP 2 and 3 are available but are not as developed as those for STEP 1. At this stage, it's better to focus on doing past papers. The accompanying topic notes for these modules are quite useful.

Mark schemes can be found on my website. It's best to attempt questions for a while before consulting solutions. I found it helpful to try for an hour, then take a break for a day if I was still stuck, and return with a fresh perspective. If you're determined to solve it yourself, consider reading slightly past where you were stuck to get a hint and see if you can continue from there.

Regarding papers, it's worthwhile to learn how to do some applied questions, preferably statistics. Many who have taken STEP wish they had worked on applied questions earlier, especially Statistics, which often turn into pure questions once the probability language is interpreted - usually combinatorics or integration. Mechanics can be challenging initially but will become clearer over time. Start with topics such as collisions, projectiles, circular motion, and simple harmonic motion, as these involve less complex modeling and help build a foundation. Gradually, you'll become more comfortable with complex modeling for other mechanics topics.

Instead of doing every paper as a timed mock initially, focus on getting proficient in a range of topics. Once you are comfortable with various topics, start doing timed papers to practice pacing.

When selecting questions, always aim for 6. It's usually easier to earn the first 5 marks of a question than the last 5, and questions are often weighted such that the final difficult part is worth a smaller portion of the total marks—this is common for graph sketches, where the initial parts involve relatively easy results that assist in drawing the graph.

For picking questions, read through and rate them out of 10 for difficulty. This way, you won't need to keep rereading during the exam, and with practice, you'll quickly identify which topics suit you best, reducing decision time to about 2 or 3 minutes.

Finally, during the exam, stay calm even if you struggle with a question. The exams are designed to test your ability to think under pressure. Mocks are valuable for getting used to this experience. If you panic, you may start making mistakes, so try to stay focused despite difficulties. Good luck with your exams!

For further guidance, refer to the document on my website about common mistakes I made for STEP; it should also be helpful for MAT.

TMUA

For Maths/CS students, the TMUA (Test of Mathematics for University Admission) is a multiple-choice exam. Although I never took it myself, it may be helpful preparation for the MAT papers, though it is easier than MAT multiple choice. There's are more resources by someone on TSR. All papers, including specimen papers, are available on the 2020 Thread which likely has updated annual versions.

The TMUA has become more relevant for those taking MAT due to its change in 2024 to have more multiple choice questions. TMUA has different content and style than MAT multiple choice though.

Miscellaneous resources

Here are some resources which aren't directly related to the interview or STEP/MAT which may be useful.

Some hard probability questions by the creator of the CSAT. These are useful for Maths and Computer Science students, but they might be a bit too difficult.

Personal Statement/Extracurriculars

First, for Maths at universities like Cambridge, Warwick, and Oxford (most high-ranking ones but not all), they won't really read or care much about your personal statement. For example, Warwick claims they briefly look at it for a few minutes and never look at it again, and I've heard Trinity have said that they read it just because they have to. This is because they have better means to measure the suitability of a candidate for their course, such as interviews and admissions tests.

What you should take away from this isn't that you don't need to write a good personal statement but that you don't need to worry about doing extremely specific extracurriculars or very minute details. Make sure the statement is good for other universities. James Munro, the admissions director at Oxford, put it best when he said, "If you have an hour to either improve your personal statement or do some maths, you should do some maths."

They're generally not interested at all in anything in your personal statement which isn't about maths, but it is good to have a line or two on this for the sake of other universities. In the interview, they're generally unlikely to ask you about anything on your personal statement, but it's worth being aware of what you wrote in case they ask you about something related to it. If you claim you know a lot of university maths, you should know it! You should also know a little bit around it too. To take an example from university maths: if you say you know Cauchy's integral theorems, then you should know Liouville's theorem, an important (but quite quick) deduction from Cauchy's integral formula.

However, this doesn't mean that doing extracurricular maths is a bad thing. In fact, it's probably the best thing you can be doing at this stage – it's just that you shouldn't do things just for the personal statement. Instead, explore the maths you enjoy the most and want to learn about. That way, you'll learn a lot more and end up organically having a lot of things to write on your personal statement later on, which will make writing it a lot easier. Here are some possible suggestions of things you can do that you might enjoy:

- 1. British Mathematical Olympiad problems. These are fun problems focusing on problem-solving ability over specific knowledge. They are difficult, but the first few questions tend to be accessible, and you'll find you improve fairly quickly. Past papers are here, and you might find books like *A Mathematical Olympiad Primer* by Geoff Smith or a more advanced book, *Problem Solving Tactics* by Angelo Di Pasquale et al., useful for Olympiad-style problems. You might also enjoy doing British Physics and British Informatics Olympiad problems if you do Physics or Computer Science, as they involve a lot of problem-solving similar to maths. There are also quite a few websites for programming problems online, such as Project Euler or competitions on websites like Kattis or Codeforces.
- 2. **STEP Foundation modules.** These are more directly relevant to what you'll need to study for the STEP exams in year 13 once you get your offer. They're a great way to ease into doing STEP questions and can introduce you to new maths you might enjoy. . Somewhat related is Stephen Siklos booklet. for STEP, which

is similar to the foundation modules.

In a similar vein, you can start looking at Oxford MAT papers, which are tamer than STEP as they're taken earlier in year 13. Oxford is also running a bunch of maths content on their YouTube channel weekly with James Munro, where students discuss some aspect of maths they enjoy:.

- 3. YouTube channels such as 3blue1brown, Michael Penn, and blackpenredpen often discuss interesting math. The latter two focus more on solving problems from various sources (Putnam, Olympiad, etc.), which you can also look into.
- 4. Reading a book about maths. This is something many people put on their personal statements. I feel like it's useful but not necessary, so don't worry about forcing yourself to mention you've read a book. There are many reading lists by universities online; here's one by Cambridge: Cambridge reading list.

I haven't read most of the books on the list. I recommend anything from the 'A Very Short Introduction To' series, as they're small enough to fit in your pocket and read while walking around. I read the one on numbers while walking to school.

- 5. **Interview preparation.** It might be a bit early to be doing this stuff, but there's a lot of fun problem-solving in interview preparation resources such as the TBO booklet.
- 6. Online courses. You can find many online math courses on websites like Coursera. One I enjoyed was this course on Fibonacci numbers by Professor Chasnov. You can find a lot of university-level topics and things that aren't taught at A-level. You can also look through university courses with notes online; Cambridge notes by Dexter Chua are well known. However, I don't recommend learning university maths yet because you have three years to do that already! Only do it if it's the most interesting thing in maths which you want to do.

Writing the Personal Statement

The personal statement was my first time writing a serious application and set the template for how I wrote all my subsequent applications - for my Masters, PhD and scholarships. I think part of the value in writing a good personal statement is getting good at writing these types of things so it is well worth the effort put in. The following advice is essentially how I constructed my personal statement.

I think it's much better to write about specific things you learned or problems you solved rather than making general comments. The easiest way I found to write a personal statement was to talk about what I'd gained from doing certain things. By being specific and going into detail, I could discuss the maths I'm interested in and what I've learned from it. For example:

"Putnam problems introduced me to more advanced problem-solving and encouraged me to bring together different areas of mathematics I'd learned, such as matrices and number theory, in problem B4 in the 1994 Putnam paper. Because this combination of subjects was unfamiliar, I worked towards a solution by gathering as much information as possible and synthesizing it. It also introduced me to a new perspective on Pell's equation, which I studied further in a paper by Seung Hyun Yang."

This example also highlights how I structured my personal statement. I tried to make each topic flow from another to show that my mathematical studies weren't in isolation—interest in one thing often led to another. This made it easier to read and less like a list of things, creating a narrative of my mathematical journey. Another benefit was that it shortened the number of characters by linking topics rather than listing them separately.

You could also discuss links between different things you discovered as you studied more math. That could be across different fields of maths (e.g., analysis and number theory to show the sum of reciprocals of primes diverges—look up the Euler product of the Riemann zeta function if you're interested) or even across subjects. I liked discussing how programming helped me solve problems by verifying things numerically or how mathematical ideas made my algorithms more efficient (e.g., using symmetry to avoid recalculating the same things). Another good subject to link is Physics, where you could solve a physical problem and encounter new maths. For me, analyzing pendulums led to encountering something called an elliptic integral.

I applied this method writing about what I gained from each activity to everything in my personal statement, focusing on what I learned or found useful. I don't think it's worth worrying about where you learned it from. Whether it's a book, a YouTube video, or an online article, Oxford and Cambridge won't care about the source, but rather what you gained from it. So don't stress if you can't find a book or if you mention a YouTube video. With bigger sources like books, you can zoom in on specific things you learned, making it more versatile.

Regarding introductions and conclusions, you should write what feels right for you. I'm not a fan of quotes or writing extensively about why you love maths. I just mentioned my interest and began discussing my studies to show it. For my conclusion, I said something about how I enjoy maths and want to pursue it at university. You don't need to end with a 'bang'; I preferred keeping it straightforward. One piece of advice I received was to link back to why I want to study maths throughout, like "this gave me a skill that's useful in math degrees," typically at the end of a paragraph. You might like doing that.

Some personal statement advice from Cambridge.